A Novel Approach for Solving the Time-Varying Complex-Valued Linear Matrix Inequality Based on Fuzzy-Parameter Zeroing Neural Network

Jiajie Luo
School of Computing
Newcastle University
Newcastle upon Tyne, UK
j.luo26@newcastle.ac.uk

Jichun Li
School of Computing
Newcastle University
Newcastle upon Tyne, UK
jichun.li@ncl.ac.uk

William Holderbaum
School of Science, Engineering and Environment
University of Salford
Manchester, United Kingdom
w.holderbaum@salford.ac.uk

Jiguang Li
The North of England Robotics Innovation Centre
University of Salford
Manchester, United Kingdom
j.li56@salford.ac.uk

***Abstract*—Solving linear matrix inequality (LMI) is crucial across diverse fields, and the emergence of zeroing neural networks (ZNN) presents a novel solution for the time-varying LMI (TV-LMI) challenge. However, the application of ZNN to solve the time-varying complex-valued LMI (TVCV-LMI) problem remains unexplored. Therefore, we introduce a novel fuzzy-parameter ZNN (FP-ZNN) model in this study to tackle the TVCV-LMI problem. With the introduction of fuzzy logic system (FLS), the FP-ZNN model is able to adjust the fuzzy convergence parameter (FCP) in a real-time manner, responding to any change in the system error and achieving the best performance. We also use an exponential activation function (EAF) in our study, which makes the FP-ZNN model fixed-time stable. To verify and illustrate the superior features of the elegant FP-ZNN model, detailed theoretical analysis, together with numerical experiments, are provided, and the results emphasize the fixed-time stability and adaptiveness of the FP-ZNN model further. As a novel approach, we provide an elegant solution to the TVCV-LMI problem in this paper.**

***Index Terms*—zeroing neural network, fuzzy logic system, linear matrix inequality, complex number**

I. Introduction

As a novel recurrent neural network, the zeroing neural network (ZNN) has been used in many applications. For example, Chen *et al.* proposed a disturbance suppression ZNN and applied it in the robust synchronization of chaotic systems [1]. Jia *et al.* applied the ZNN approach to solve the time-variant QP problem [2]. In [3], an elegant ZNN model is proposed by Dai *et al.*, and they applied it in the dynamic positioning problem [3]. ZNN models have also been used in control applications, and Ma *et al.* proposed a novel ZNN control strategy for mobile robot manipulators [4]. In [5], Kovalnogov *et al.* proposed a novel image color restoration algorithm based on ZNN. ZNN was first used in the field of scientific computing to solve time-varying equations, and then was extended to solve practical problems.

ZNNs have proven to be a powerful tool in addressing the time-varying linear matrix inequality (TV-LMI) problem. Notably, there are various instances where ZNN has been applied to tackle TV-LMI challenges. For instance, a noise-tolerant ZNN [6] is designed specifically for addressing the TV-LMI problem. In [7], a series of ZNN models featuring novel activation functions is employed to solve the TV-LMI problem within fixed-time constraints. Additionally, a varying-parameter ZNN model is explored in [8] to effectively solve the TV-LMI problem. While the complex-valued linear matrix inequality (LMI) problem represents a distinct branch within the broader LMI problems [9]–[11], there has been a noticeable gap in research concerning the potential application of ZNN models to solve the time-varying complex-valued LMI (TVCV-LMI) problem.

There are various engineering challenges that involve discussions in the complex domain, such as digital signal processing, classical control theory, and image processing. Complex numbers play a crucial role in understanding the world. However, it is worth mentioning that most traditional ZNNs are only capable of handling problems defined in the real domain. To fill in the gaps, a variety of complex-valued ZNN models are proposed recently [12]–[14]. In the literature, there are mainly two kinds of complex-valued ZNN models. In the first one, activation will be applied to the real and imaginary parts of complex numbers [12], while in the second one, activation is only applied to the magnitudes of complex numbers [13]. At present, the state-of-the-art complex ZNNs have only been used in solving equations [14]. It is important to note that there are very few reported ZNNs by researchers for solving complex-valued inequalities.

With the development of technology, researchers have proposed a series of variants of ZNN, including fixed-time convergent ZNN [15], finite-time ZNN [16] and noise-tolerant ZNN

[17]. Among these elegant models, fuzzy-parameter ZNN (FP-ZNN) represents an important area of research [18]–[20]. In a traditional ZNN model, the convergence parameter won't change as the error changes. However, designing zeroing neural networks with variable convergence parameter to achieve adaptive convergence is beneficial. As a result, researchers have developed various FP-ZNN models which convergence parameter can be adjusted as the error changes. FP-ZNN models are widely used. For instance, Kong *et al.* introduced a ZNN model with fuzzy parameters for the cooperative control of multiple redundant manipulators [18]. In comparison to the traditional ZNN model, the FP-ZNN model is more intelligent. The convergence parameters in traditional ZNN models are fixed while the fuzzy convergence parameters in FP-ZNN models is adjustable. According to the system error, the fuzzy convergence parameters will be adjusted in real time.

In this paper, we proposed a novel FP-ZNN model to solve the TVCV-LMI problem in a fixed time. In Section II, we introduce some necessary lemmas and definitions. Next, the elegant FP-ZNN model is described in Section III. Together with the theoretical analysis, which is shown in Section V, the numerical experiments in Section VI verify the proposed model further. Finally, we summarize our study in Section VII.

II. Preliminaries

In this section, we will introduce the complex matrix theory and the time-varying complex-valued linear matrix inequality (TVCV-LMI) problem.

A. Complex analysis

As an extension of real numbers, complex numbers are widely used in both science and engineering. In general, we can express a complex number in the following form:

$$c = a + b\mathbf{i}. \tag{1}$$

Here, we call a the real part of the complex number c, b the imaginary part of the complex number c, and $\mathbf{i}$ is the unit imaginary number.

It is possible to represent complex numbers by real matrices, and we describe such conversion as follows.

Definition 1 *We can represent a complex number $c = a + b\mathbf{i}$ by a real matrix:*

$$\hat{c} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}. \tag{2}$$

Here, we use $\hat{c}$ to represent the corresponding real matrix of the complex number c.

Definition 2 *We can represent a complex matrix $C = A + B\mathbf{i}$ by a real matrix:*

$$\hat{C} = \begin{pmatrix} A & -B \\ B & A \end{pmatrix}. \tag{3}$$

Here, we use $\hat{C}$ to represent the corresponding real matrix of the complex matrix C.

In the following discussion, we give some analytical properties of the above representation [21]. Supposing X is a complex matrix, we will use $\hat{X}$ to denote the real representation of X.

Lemma 1 *For addition, supposing X, Y and S are complex matrices, we can obtain:*

$$S = X + Y \Leftrightarrow \hat{S} = \hat{X} + \hat{Y}. \tag{4}$$

Lemma 2 *For multiplication, supposing X, Y and P are complex matrices, we can obtain:*

$$P = M \times N \Leftrightarrow \hat{P} = \hat{M} \times \hat{N}. \tag{5}$$

B. Time-Varying Complex-Valued Linear Matrix Inequalities

In this section, we introduce the time-varying complex-valued linear matrix inequalities (TVCV-LMI) problem.

We denote $A(t)$, $B(t)$ and $X(t)$ by three time-varying matrices, then the TVCV-LMI problem can be expressed as follows:

$$A(t)X(t) \leq B(t). \tag{6}$$

Here, $X(t)$ is unknown.

Based on Lemma 2, (6) can also be expressed as follows:

$$\hat{A}(t)\hat{X}(t) \leq \hat{B}(t). \tag{7}$$

III. Zeroing Neural Networks

In this part, the FP-ZNN model will be described in detail. To help readers understand it in a better way, we will introduce the traditional neural networks first.

A. Traditional Zeroing Neural Network

Based on (7), we can define the following residual function:

$$\mathcal{R}(t) = \hat{A}(t)\hat{X}(t) - \hat{B}(t). \tag{8}$$

It is obvious that (8) cannot be served as an error function since a solution $\hat{A}(t)\hat{X}(t) - \hat{B}(t) < 0$ to the TVCV-LMI problem doesn't satisfy $\mathcal{R}(t) = 0$. But we can construct the error function based on (8) and get:

$$\mathcal{E}(t) = (\max\{\mathbf{0}, \mathcal{R}(t)\})^2/2. \tag{9}$$

Here, the zero matrix $\mathbf{0}$ is in the same shape of $\mathcal{R}(t)$, and $\max\{\cdot,\cdot\}$ is the max function. For a solution $\hat{A}(t)\hat{X}(t) - \hat{B}(t) < 0$ to the TVCV-LMI problem, the $\mathcal{E}(t) = 0$ now.

There are a variety of design formulas for ZNN models, and we choose the original ZNN formula to build the traditional ZNN (TZNN) model:

$$\dot{\mathcal{E}}(t) = -\mu\Phi(\mathcal{E}(t)). \tag{10}$$

Here, $\Phi(\cdot)$ is an activation function, and we call the constant μ the convergence parameter (CP).

Substituting (9) into (10), we obtain the final model customized for the TVCV-LMI problem:

$$\max\{\mathbf{0}, \mathcal{R}(t)\}\dot{\mathcal{R}}(t) = -\frac{\mu}{2}\max\{\mathbf{0}, \mathcal{R}(t)\}^2, \tag{11}$$

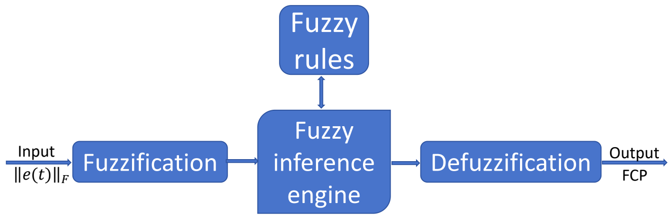

Fig. 1. The diagram of fuzzy logic system.

where $\mathbf{0}$ and $\mathcal{R}(t)$ are the zero matrix and the residual matrix respectively.

It is obvious that (11) is equivalent to:

$$\dot{\mathcal{R}}(t) = -\frac{\mu}{2}\max\{\mathbf{0}, \mathcal{R}(t)\}. \tag{12}$$

In the following analysis, we will use the concept of settling-time function, and the definition is as follows:

Definition 3 *We define the settling-time function as $\mathcal{T}(X(0))$, which means that the error function $\mathcal{E}(t)$ is supposed to converge in $\mathcal{T}(X(0))$. Here, $X(0)$ is the initial state.*

B. Fuzzy-Parameter Zeroing Neural Network

Normally, the CP in a TZNN model (10) is a constant, ant it won't change as the error changes. In this section, we designed a fuzzy-parameter zeroing neural network (FP-ZNN), which CP is generated by the fuzzy logic system (FLS) and can be adjusted according to the feedback of error, to solve the TVCV-LMI problem.

In the following discussion, we describe the components of FLS in detail.

1) *Fuzzification* is the process to transform scalar values into fuzzy sets. Membership functions are essential in representing fuzzy sets, and we will use the following membership functions in our study:

   a. Triangle function:

$$m(y) = \begin{cases} 0 & \text{if } y \le p_1, \\ \frac{y-p_1}{p_2-p_1} & \text{if } p_1 \le y < p_2, \\ \frac{p_3-y}{p_3-p_2} & \text{if } p_2 \le y < p_3, \\ 0 & \text{if } y \ge p_3. \end{cases} \tag{13}$$

   Here, p_1, p_2 and p_3 are parameters.

   b. Gaussian function:

$$m(y) = \exp(-\frac{(y-q_1)^2}{2q_2^2}). \tag{14}$$

   Here, $\exp(\cdot)$ is the exponential function, q_1 and q_2 denote parameters.

   Specifically, the triangular membership function and the Gaussian membership function are used in the fuzzification and defuzzification process respectively. To visualize these membership functions, we plot them in Fig. 2 and Fig. 3 respectively. In particular, we set the output range $[O_l,\ O_r]$ of FCP as $[O_l,\ O_r] = [1,\ 10]$.

2) *Inference* is the technique to get suitable output fuzzy sets after an evaluation of the input fuzzy sets. Fuzzy rules, which can be described through a series of IF-THEN statements, will be used in the inference process. Specifically, we use the following fuzzy rules in this study:

   F_1: if $E = O$, then $\zeta = O$,
   F_2: if $E = S$, then $\zeta = S$,
   F_3: if $E = L$, then $\zeta = L$.

   Here, L, S and O are the fuzzy sets of large, small, and zero values respectively, E and ζ represent the error and FCP respectively.

   With the fuzzy rules F_1, F_2 F_3 and the union operator $\cup$, $F = F_1 \cup F_2 \cup F_3$ is defined to obtain $\zeta = E \circ F$, where E is the fuzzy set input to FLS, and $\circ$ denotes the process of fuzzy inference.

3) *Defuzzification* is the process to transform fuzzy sets into scalar values. To achieve defuzzification, the traditional centroid method is used in this study, and it can be expressed as:

$$\zeta = \frac{\int_\zeta \zeta \mathcal{R}_{E\circ F}(\zeta)d\zeta}{\int_\zeta \mathcal{R}_{E\circ F}(\zeta)d\zeta}. \tag{15}$$

   Here, $\mathcal{R}_{E\circ F}(\zeta) = \mathcal{R}_{E\circ F_1} \vee \mathcal{R}_{E\circ F_2} \vee \mathcal{R}_{E\circ F_3}$, $\mathcal{R}_{E\circ F} = \mathcal{R}_E \wedge \mathcal{R}_F$. $\wedge$ and $\vee$ are the minimum and maximum operators respectively.

A diagram of the FLS used in this study is presented in Fig. 1. The fuzzification module converts crisp input values into a series of fuzzy sets. The fuzzy inference engine accepts these output fuzzy sets from the fuzzification module and then generate suitable fuzzy sets according to the built-in fuzzy rules. Finally, the defuzzification module will defuzzify these fuzzy sets and output the final FCP.

With the FCP $\zeta(t)$ and a activation function $\Phi(\cdot)$, the fuzzy-parameter zeroing neural network (FP-ZNN) can be described as follows:

$$\dot{\mathcal{E}}(t) = -\zeta(t)\Phi(\mathcal{E}(t)). \tag{16}$$

In this study, the activation function $\Phi(\cdot)$ used in the proposed FA-ZNN model is specified as the following exponential-power activation function (EAF):

$$\Phi(x) = a_1 \exp(|x|^p)|x|^{1-p}\text{sign}(x)/p, \tag{17}$$

where $\text{sign}(x)$ denotes the sign function, $a_1 > 0$ and $0 < p < 1$ are parameters.

In addition to EAF (17), the following activation functions may also be used in this study:

1) Linear activation function (LAF):

$$\Phi(x) = a_1 y, \tag{18}$$

2) Power activation function (PAF):

$$\Phi(y) = a_1 |y|^p \,\text{sign}(y), \tag{19}$$

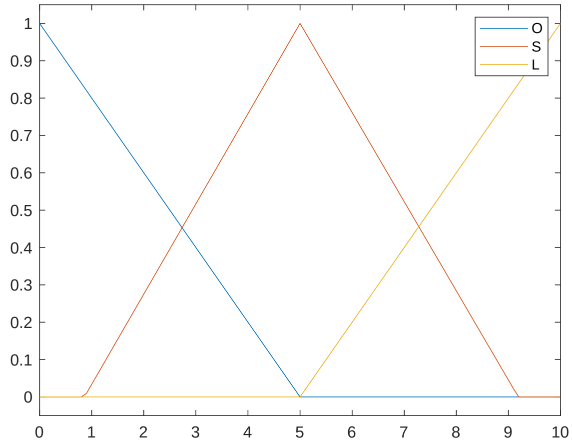

Fig. 2. The triangle membership function.

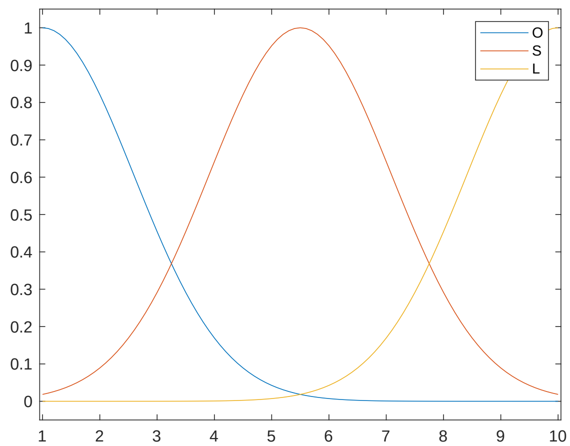

Fig. 3. The Gaussian membership function.

3) Bi-power activation function (BAF):

$$\Phi(y) = (a_1 |y|^p + a_2 |y|^q)\text{sign}(y), \tag{20}$$

where $a_1 > 0, a_2 > 0, 0 < p < 1$ and $q > 1$ are parameters.

IV. Theoretical Analysis

In this section, we will prove that the proposed FP-ZNN model (16) is Lyapunov stable, fixed-time convergent.

A. Lyapunov Stability

Theorem 1 *If FP-ZNN model* (16) *with EAF* (17) *is used to solve the TVCV-LMI problem, then the system will be Lyapunov stable.*

Proof *As stated before, we set the output range* $[O_l,\ O_r]$ *of FCP* $\zeta(t)$ *as* $[O_l,\ O_r] = [1,\ 10]$. *It is obvious that*

$$\begin{aligned}\mathcal{E}(t) &= (\max\{0, \mathcal{R}(t)\})^2/2 \geq 0,\\ \zeta(t) &> O_l = 1,\\ \Phi(x) &\geq 0, \quad \forall x > 0.\end{aligned} \tag{21}$$

Therefore, for each element $e_{ij}(t)$ *of the error matrix* $\mathcal{E}(t)$, *we have:*

$$\zeta(t)e_{ij}(t)\phi(e_{ij}(t)) = \begin{cases} > 0 & \text{if } e_{ij}(t) \neq 0 \\ = 0 & \text{if } e_{ij}(t) = 0 \end{cases}. \tag{22}$$

We can define Lyapunov functions $l_{ij}(t) = \frac{e_{ij}^2(t)}{2}$ *for each element* $e_{ij}(t)$ *of the error matrix* $\mathcal{E}(t)$, *then for* $i = 1\ldots n,\ j = 1\ldots m$, *we have:*

$$\begin{aligned}\dot{l}_{ij}(t) &= e_{ij}(t)\dot{e}_{ij}(t),\\ &= -\zeta(t)e_{ij}(t)\phi(e_{ij}(t)),\\ &\leq 0.\end{aligned} \tag{23}$$

Here, $\phi(x)$ *is the activation function, and* $\zeta(t)$ *denotes the FCP* (16). *From* (16), *we know* $\dot{e}_{ij}(t) = -\zeta(t)e_{ij}(t)$.

It is clear that the system is Lyapunov stable based on Lyapunov's theorem.

B. Fixed-Time Convergence

Theorem 2 *For FP-ZNN model* (16) *with any initial state* $X(0)$, *if EAF* (17) *is used, then the error will converge within:*

$$\mathcal{T}(X(0)) \leq \frac{1}{a_1 O_l}, \tag{24}$$

where $a_1 > 0$ *is the predefined parameter in EAF* (17), *and* O_l *is the minimum value of the FCP* $\zeta(t)$.

Proof *We can build* $g_{ij}(t) = |e_{ij}(t)|$ *to obtain:*

$$\begin{aligned}\dot{g}_{ij}(t) &= \dot{e}_{ij}(t)\text{sign}(e_{ij}(t)),\\ &= -\zeta(t)\phi(e_{ij}(t))\text{sign}(e_{ij}(t)),\\ &= -a_1\zeta(t)\exp(|e_{ij}(t)|^p)|e_{ij}(t)|^{1-p}/p,\\ &\leq -a_1 O_l \exp(|e_{ij}(t)|^p)|e_{ij}(t)|^{1-p}/p,\\ &\leq -a_1 O_l \exp(g_{ij}^p(t))g_{ij}^{1-p}(t)/p\end{aligned} \tag{25}$$

where $a_1 > 0$ *and* $0 < p < 1$ *are predefined parameters, and* O_l *is the minimum value of the FCP* $\zeta(t)$.

By solving the inequality equation (25), *we have*

$$\begin{aligned}T_{ij}(X(0)) &\leq \frac{1 - \exp(-g_{ij}^p(0))}{a_1 O_l},\\ &= \frac{1 - \exp(-|e_{ij}(0)|^p)}{a_1 O_l},\\ &\leq \frac{1}{a_1 O_l}.\end{aligned} \tag{26}$$

It is obvious that for the error matrix $\mathcal{E}(t)$ *as a whole, we have:*

$$\begin{aligned}\mathcal{T}(X(0)) &= \max_{i=1\ldots n, j=1\ldots m}\{T_{ij}\},\\ &\leq \frac{1}{a_1 O_l},\end{aligned} \tag{27}$$

Here, O_l *is the minimum value of the FCP* $\zeta(t)$, $a_1 > 0$ *and* $0 < p < 1$ *denote parameters, and* T_{ij} *is the settling time for the* ij*th element* $e_{ij}(t)$ *of the error.*

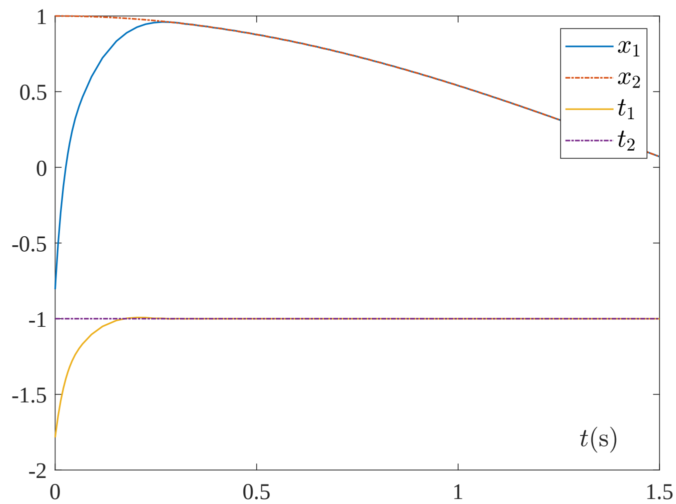

Fig. 4. The trajectories of the unknown complex number $x(t) = x_1(t) + x_2(t)\mathbf{i}$ in Example 1.

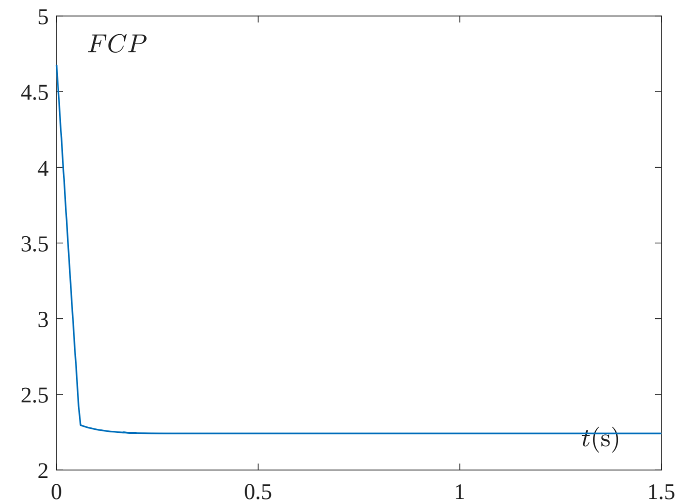

Fig. 6. The trajectories of FCP in Example 2.

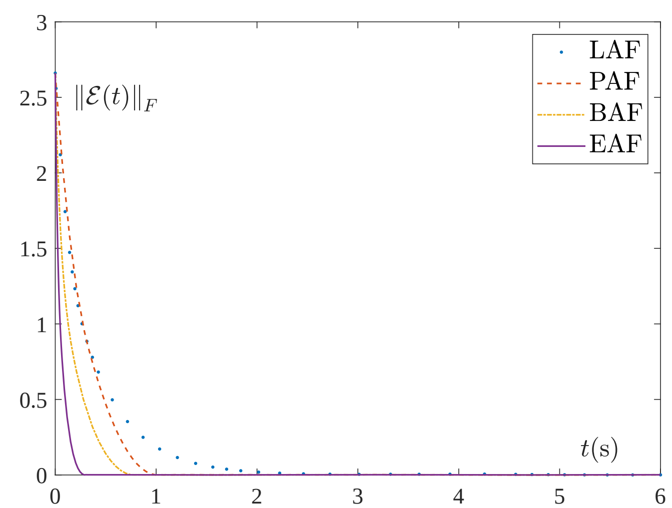

Fig. 5. The trajectories of error in Example 1.

V. Numerical Experiments

In this section, two experiments are provided to verify the fixed-time convergence and the adaptiveness of the proposed FA-ZNN model.

A first-order TVCV-LMI problem (6) is considered in following experiments, which can be described as:

$$(\sin(t) + \mathbf{i})x(t) \leq -1 + \cos(t)\mathbf{i}. \quad (28)$$

Here, $x(t) = x_1(t) + x_2(t)\mathbf{i}$ is the unknown complex number to be solved, and $\mathbf{i}$ is the imaginary unit.

We denote $\hat{X}(t)$ by the real representation of $x(t) = x_1(t) + x_2(t)\mathbf{i}$. By Definition 1, we can transform (28) into:

$$\begin{pmatrix} \sin(t) & -1 \\ 1 & \sin(t) \end{pmatrix} \hat{X}(t) \leq \begin{pmatrix} -1 & -\cos(t) \\ \cos(t) & -1 \end{pmatrix}. \quad (29)$$

A. Fixed-time Convergence

Example 1 *In this example, we will demonstrate the fixed-time convergence of the proposed model. The EAF* (17) *and other three activation functions (LAF* (18)*, PAF* (19)*, and SAF* (20)*) will also be used in this example. We set the common parameters as* $a_1 = a_2 = a_3 = a_4 = 1, p = 0.5$*. Based on Theorem 2, we can obtain that:*

$$\begin{aligned} \mathcal{T}(X(0)) &\leq \frac{1}{a_1 O_l}, \\ &= \frac{1}{1 \times 1}, \\ &\approx 1. \end{aligned} \quad (30)$$

With EAF (17)*, we show the trajectories of* $x(t)$ *in Fig. 4. Here,* $t_1(t)$ *and* $t_2(t)$ *denote the theoretical solutions of* $x_1(t)$ *and* $x_2(t)$ *respectively. With other three activation functions, we also show the trajectories of error in Fig. 5. It is obvious that, with* (17)*, the error converged within* 1 *seconds, which verify the theoretical results further. We can also see from the figure that FP-ZNN model* (16) *with EAF* (17) *converged faster that FP-ZNN model* (16) *with other three activation functions*

B. Adaptiveness

Example 2 *Under the control of FP-ZNN model* (16) *with EAF* (17)*, the trajectories of FCP are shown in Fig. 6. As the error decreases, the FCP* (16) *can dynamically adjust its convergence, which is more intelligent than the traditional ZNN model. The convergence parameters in traditional ZNN models are usually fixed and cannot be adjusted.*

VI. Conclusion

In this study, we proposed a novel FP-ZNN model to solve the TVCV-LMI problem. We demonstrated that the proposed FP-ZNN model is adaptive and fixed-time convergent. Together with detailed theoretical analysis, numerical experiments verified the superior features further. This study, as a novel approach, provides a elegant solution to the TVCV-LMI problem. However, the FP-ZNN models still have limitations. Compared with the traditional ZNN models, the introduction of fuzzy logic system brings a great computational burden.

REFERENCES

[1] W. Chen, J. Jin, C. Chen, F. Yu, and C. Wang, "A disturbance suppression zeroing neural network for robust synchronization of chaotic systems and its FPGA implementation," *Int. J. Bifurcation Chaos*, vol. 32, no. 14, p. 2250210, 2022.

[2] L. Jia, L. Xiao, J. Dai, Z. Qi, Z. Zhang, and Y. Zhang, "Design and application of an adaptive fuzzy control strategy to zeroing neural network for solving time-variant qp problem," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 6, pp. 1544–1555, 2021.

[3] J. Dai, Y. Li, L. Xiao, and L. Jia, "Zeroing neural network for time-varying linear equations with application to dynamic positioning," *IEEE Transactions on Industrial Informatics*, vol. 18, no. 3, pp. 1552–1561, 2022.

[4] Z. Ma, S. Yu, Y. Han, and D. Guo, "Zeroing neural network for bound-constrained time-varying nonlinear equation solving and its application to mobile robot manipulators," *Neural Computing and Applications*, vol. 33, no. 21, pp. 14231–14245, 2021.

[5] V. N. Kovalnogov, R. V. Fedorov, D. A. Demidov, M. A. Malyoshina, T. E. Simos, V. N. Katsikis, S. D. Mourtas, and R. D. Sahas, "Zeroing neural networks for computing quaternion linear matrix equation with application to color restoration of images," *AIMS Math*, vol. 8, no. 6, pp. 14321–14339, 2023.

[6] L. Xiao, W. Song, X. Li, L. Jia, J. Sun, and Y. Wang, "Design and analysis of a noise-resistant ZNN model for settling time-variant linear matrix inequality in predefined-time," *IEEE Trans. Ind. Inf.*, vol. 18, no. 10, pp. 6840–6847, 2021.

[7] Y. Zeng, L. Xiao, K. Li, J. Li, K. Li, and Z. Jian, "Design and analysis of three nonlinearly activated ZNN models for solving time-varying linear matrix inequalities in finite time," *Neurocomputing*, vol. 390, pp. 78–87, 2020.

[8] L. Xiao and W. Song, "Super finite-time variable parameter ZNN models for time-variant linear matrix inequality," *J. Franklin Inst.*, vol. 359, no. 4, pp. 1640–1654, 2022.

[9] MathWorks, "Advanced LMI techniques."

[10] StephenBoyd, LaurentElGhaoui, and E. andV enkataramanan Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. London: Society for Industrial and Applied Mathematics, 1994.

[11] A. Ben-Israel, "Linear equations and inequalities on finite dimensional, real or complex, vector spaces: A unified theory," *J. Math. Anal. Appl.*, vol. 27, no. 2, pp. 367–389, 1969.

[12] L. Xiao, L. Li, J. Tao, and W. Li, "A predefined-time and anti-noise varying-parameter ZNN model for solving time-varying complex stein equations," *Neurocomputing*, vol. 526, pp. 158–168, 2023.

[13] J. Dai, Y. Li, L. Xiao, L. Jia, Q. Liao, and J. Li, "Comprehensive study on complex-valued ZNN models activated by novel nonlinear functions for dynamic complex linear equations," *Inform. Sci.*, vol. 561, pp. 101–114, 2021.

[14] Y. He, L. Xiao, F. Sun, and Y. Wang, "A variable-parameter ZNN with predefined-time convergence for dynamic complex-valued lyapunov equation and its application to aoa positioning," *Appl. Soft Comput.*, vol. 130, p. 109703, 2022.

[15] J. Jin, J. Zhu, J. Gong, and W. Chen, "Novel activation functions-based znn models for fixed-time solving dynamirc sylvester equation," *Neural Computing and Applications*, vol. 34, no. 17, pp. 14297–14315, 2022.

[16] L. Xiao, L. Jia, Y. Wang, J. Dai, Q. Liao, and Q. Zhu, "Performance analysis and applications of finite-time znn models with constant/fuzzy parameters for tvqpei," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 33, no. 11, pp. 6665–6676, 2021.

[17] Z. Hu, L. Xiao, J. Dai, Y. Xu, Q. Zuo, and C. Liu, "A unified predefined-time convergent and robust znn model for constrained quadratic programming," *IEEE Transactions on Industrial Informatics*, vol. 17, no. 3, pp. 1998–2010, 2020.

[18] Y. Kong, S. Chen, Y. Jiang, H. Wang, and H. Chen, "Zeroing neural network with fuzzy parameter for cooperative manner of multiple redundant manipulators," *Expert Systems with Applications*, vol. 212, p. 118735, 2023.

[19] V. N. Katsikis, P. S. Stanimirović, S. D. Mourtas, L. Xiao, D. Stanujkić, and D. Karabašević, "Zeroing neural network based on neutrosophic logic for calculating minimal-norm least-squares solutions to time-varying linear systems," *Neural Processing Letters*, vol. 55, no. 7, pp. 8731–8753, 2023.

[20] L. Jia, L. Xiao, J. Dai, and Y. Cao, "A novel fuzzy-power zeroing neural network model for time-variant matrix moore–penrose inversion with guaranteed performance," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 9, pp. 2603–2611, 2020.

[21] J. Daboul and R. Delbourgo, "Matrix representation of octonions and generalizations," *J. Math. Phys.*, vol. 40, no. 8, pp. 4134–4150, 1999.

[22] L. Xiao, Y. Cao, J. Dai, L. Jia, and H. Tan, "Finite-time and predefined-time convergence design for zeroing neural network: Theorem, method, and verification," *IEEE Transactions on Industrial Informatics*, vol. 17, no. 7, pp. 4724–4732, 2020.