Real-time Validation of Euler Angles Using IMU Data Fusion in Robotic Applications

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Abstract

Euler angles are of high importance for robots with 3D motions including legged, humanoid, aerial (as described in [1]), and marine robots. These angles can be obtained using data from IMUs, consisting of different sensors. In this paper, roll and pitch angles will be obtained using a complementary filter which fuses data from an accelerometer and a gyroscope. Yaw angle is calculated based on the obtained roll and pitch angles and data from a magnetometer (compass). Since one of the sensors can experience noise, a novel real-time factor is provided to measure the accuracy of obtained angles and detect sensor failures.

Different studies have evaluated and validated Euler angles, but none provide real-time capabilities except for [2]. It devised a method that utilises seven approaches to calculate the Euler angles, each using a different subset of the six accelerometer and magnetometer measurement components. This method improves fault tolerance and sensor/IMU failure detection through diverse redundancy. However, this study did not offer a method to compare the data separately obtained from the accelerometer and magnetometer. Additionally, it did not utilise a gyroscope which is typically a robust sensor against vibration noise. The current work introduces a real-time factor based on the data separately obtained from a magnetometer and a set of accelerometer and gyroscope to indicate the error of the obtained Euler angles for the subset of 3-2-1.

Ideally, Euler angles can be obtained using a 3-2-1 rotation matrix (yaw-pitch-roll which is aerospace standard sequence [2]) and the gravity components reported by an accelerometer, as shown below:

$$\boldsymbol{R_i^b} = \boldsymbol{R_z}(\psi)\,\boldsymbol{R_{y'}}(\theta)\,\boldsymbol{R_{x''}}(\phi) = \begin{bmatrix} C_\psi\, C_\theta & C_\psi\, S_\phi\, S_\theta\, -\, C_\phi\, S_\psi & S_\phi\, S_\psi +\, C_\phi\, C_\psi\, S_\theta \\ C_\theta\, S_\psi & C_\phi\, C_\psi +\, S_\psi\, S_\phi\, S_\theta & C_\phi\, S_\psi\, S_\theta\, - C_\psi\, S_\phi \\ -\, S_\theta & C_\theta\, S_\phi & C_\phi\, C_\theta \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} g_{xb} \\ g_{yb} \\ g_{zb} \end{bmatrix} = \boldsymbol{R_b^i} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \Rightarrow \begin{bmatrix} g_{xb} \\ g_{yb} \\ g_{zb} \end{bmatrix} = \begin{bmatrix} -g\,sin\,\theta \\ g\,cos\,\theta\,sin\,\phi \\ g\,cos\,\theta\,cos\,\phi \end{bmatrix} \tag{2}$$

$$\Rightarrow \phi_{accelerometer} = \arctan\frac{g_{yb}}{g_{zb}} \quad (3)$$

$$\Rightarrow \theta_{accelerometer} = \sin^{-1}\left(\frac{g_{xb}}{\sqrt{{g_{xb}}^2 + {g_{yb}}^2 + {g_{zb}}^2}}\right) \quad (4)$$

Where ϕ, θ, ψ represent roll, pitch and yaw angels which are rotations around the $\boldsymbol{x}''$, $\boldsymbol{y}'$ and $\boldsymbol{z}$ axes, respectively. $\boldsymbol{R}_i^b$ is a rotation matrix that transforms the expression of a vector in the inertial frame to the body frame (without rotating the vector). Vectors $[g_{xb} \quad g_{yb} \quad g_{zb}]^T$ and $[0 \quad 0 \quad g = 9.8]^T$ are gravity vectors expressed in the body and inertial frames, respectively.

Additionally, another relation exists between the vector of gravity expressed in the body and inertial frame.

$$\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = \boldsymbol{R}_i^b \begin{bmatrix} g_{xb} \\ g_{yb} \\ g_{zb} \end{bmatrix} \quad (5)$$

From the first and second line of this equation, $\tan\psi$ can be obtained as:

$$\tan\psi_{accelerometer} = \frac{-g_{zb}\cos\phi\sin\theta - g_{yb}\sin\phi\ \sin\theta - g_{xb}\cos\theta}{g_{zb}\ \sin\phi - \ g_{yb}\cos\phi} \quad (6)$$

$$\tan\psi_{accelerometer} = \frac{g_{zb}\ \sin\phi - \ g_{yb}\cos\phi}{g_{zb}\cos\phi\sin\theta + g_{yb}\sin\phi\ \sin\theta + g_{xb}\cos\theta} \quad (7)$$

According to equations 6 and 7, $\tan\psi_{accelerometer} = \frac{-1}{\tan\psi_{accelerometer}}$, which is a contradictory result. Therefore, ψ of the rotation sequence of 3-2-1 cannot be calculated just using an accelerometer.

However, in most applications, there are some unmodelled vibrations and disturbances which significantly affect accelerometers. Fig.1, illustrates the noise on an accelerometer mounted on a quadrotor with motors on (typical brushless motors on quadrotors can rotate up to 11000 RPM).

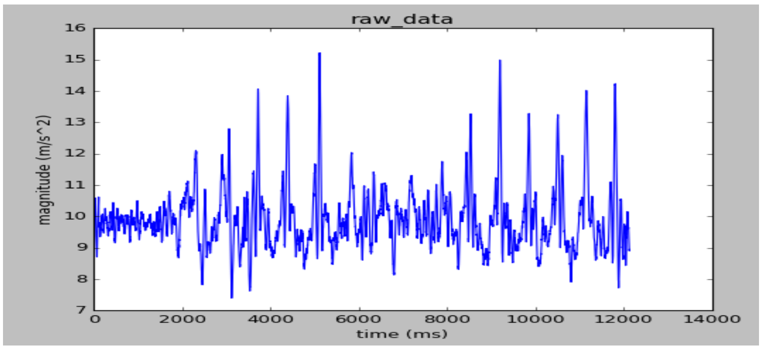

Fig. 1. Raw data of accelerometer ADXL345 under vibration noise of the quadrotor motors

Typically, gyroscopes are more robust against vibration noises than accelerometers. Gyroscopes report the current angular speed as $[p \quad q \quad r]^T$ which is the vector of the angular speed expressed in the body frame. The relation between this velocity and the rate of change of Euler angles is as follows:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{R}_i^{b^T} (\dot{\psi} \boldsymbol{k} + \dot{\theta}\, \boldsymbol{j}' + \dot{\phi}\, \boldsymbol{i}'') = \mathbf{R}_i^{b^T} \left(\dot{\psi} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \boldsymbol{R_Z}(\psi)\, \dot{\theta} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \boldsymbol{R_Z}(\psi)\, \boldsymbol{R_Y}(\theta)\, \dot{\phi} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) \tag{8}$$

$$\Rightarrow \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & sin\phi\, tan\theta & cos\phi\, tan\theta \\ 0 & cos\, \phi & -\, sin\phi \\ 0 & sin\phi\, sec\theta & sec\theta\, cos\phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{9}$$

According to equation 9, if ϕ and θ of the *n-1* th loop are known, then $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ of the nth loop can be obtained based on the raw data (p,q and r) measured by the gyroscope at the *n*th loop.

However, integrating the velocity of Euler angles with respect to time results in large drift errors as shown in Fig.2.

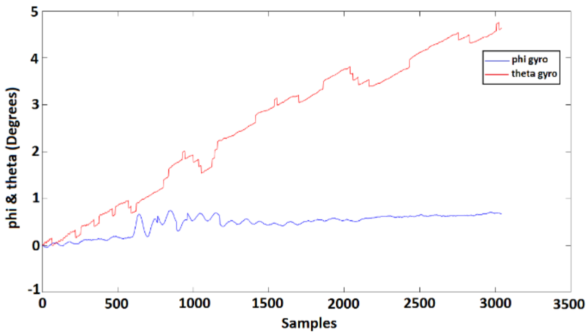

Fig. 2. Drift error of angle ϕ and θ of gyroscope L3G4200D in a stationary state

To use the advantage of both the accelerometer (removing drift errors) and gyroscope (robustness against vibration noises), the complementary filter (discussed in [3]) is used in what follows:

$$\phi_{Comp(n)} = (1-\alpha)\big(\phi_{Comp(n-1)} + \dot{\phi}_{gyro}\, dt\big) + \alpha\, \phi_{accelerometer} \tag{10}$$

$$\theta_{Comp(n)} = (1-\alpha)\big(\theta_{Comp(n-1)} + \dot{\theta}_{gyro}\, dt\big) + \alpha\, \theta_{accelerometer} \tag{11}$$

Where $\phi_{accelerometer}$ is the ϕ angle (roll) calculated via equation 3 using the accelerometer, and $\dot{\phi}_{gyro}$ is the angular velocity of ϕ angle obtained according to equation 9 using the output of the gyroscope. Additionally, the parameter dt is the time between sampling *n* and *n-1*, $\phi_{Comp(n)}$ is the roll angle calculated by the complementary filter in the

nth loop. Parameter α is the accelerometer coefficient, normally chosen as a value between 0.01 and 0.07. It indicates a reliance of 1% to 7% on the accelerometer (using equation 3) and 99% to 93% reliance on the sum of the previous angle and integration of the gyroscope velocity measured at the current loop. It can be mathematically proven that if the gyroscope and accelerometer remain stationary after experiencing some movements, the complementary angle tends to converge to the angle obtained by the accelerometer, which is the accurate value in stationary situations. Similarly, the same procedure applies for θ.

So far, angles ϕ and θ have been obtained. Now, the angle ψ (yaw) can be calculated using a compass. The magnetic field of the Earth is a 3D vector. Vector $[M_{xb} \quad M_{yb} \quad M_{zb}]^T$ is the normalised magnetic field expressed in the body frame measured by the compass. Vector $[M_{xi} \quad M_{yi} \quad M_{zi}]^T$ is the normalised magnetic field expressed in the inertial frame measured by the compass when all the Eulerian angles are zero. The reason for using normalised vectors is that the magnitude of the magnetic field of the Earth can vary at different altitudes.

$$\begin{bmatrix} M_{xi} \\ M_{yi} \\ M_{zi} \end{bmatrix} = \boldsymbol{R}_i^b \begin{bmatrix} M_{xb} \\ M_{yb} \\ M_{zb} \end{bmatrix} \Rightarrow \begin{bmatrix} M_{xi} \\ M_{yi} \\ M_{zi} \end{bmatrix}$$

$$= \begin{bmatrix} M_{zb}(sin\phi\ sin\psi\ +\ cos\phi\ cos\psi\ sin\theta)\ -\ M_{yb}(cos\,\phi\ sin\psi\ -\ cos\,\psi\ sin\phi\ sin\theta)\ +\ M_{xb}\,cos\,\psi\ cos\theta) \\ -\,M_{zb}(cos\,\psi\ sin\phi\ -cos\,\phi\ sin\psi\ sin\,\theta) + M_{yb}(cos\,\phi\ cos\,\psi\ +\ sin\phi\ sin\psi\ sin\,\theta) +\ M_{xb}cos\theta\ sin\psi \\ M_{zb}\ cos\,\phi\ cos\,\theta\ +\ M_{yb}\ cos\,\theta\ sin\,\phi -\ M_{xb}\ sin\,\theta \end{bmatrix} \quad (12)$$

Therefore, ψ can be obtained based on ϕ_{Comp} and θ_{Comp} calculated in equations 10 and 11, respectively, as follows:

$$cos\psi = \frac{M_{yi}\big(M_{zb}sin\phi -\ M_{yb}\ cos\,\phi\big) - M_{xi}\big(M_{zb}\ cos\,\phi\ sin\,\theta +\ M_{yb}\ sin\phi\ sin\theta +\ M_{xb}cos\theta\big)}{A} \quad (13)$$

$$sin\psi = \frac{M_{xi}\big(-M_{zb}sin\phi + M_{yb}\ cos\,\phi\big) - M_{yi}(M_{zb}cos\phi sin\theta + M_{yb}sin\phi\ sin\theta + M_{xb}cos\theta)}{A} \quad (14)$$

Where A is:

$$A = -\big(M_{zb}cos\phi sin\theta + M_{yb}sin\phi\ sin\theta + M_{xb}cos\theta\big)\big(M_{zb}\ cos\,\phi\ sin\,\theta +\ M_{yb}\ sin\phi\ sin\theta +\ M_{xb}cos\theta\big) + \big(M_{zb}sin\phi -\ M_{yb}\ cos\,\phi\big)(-M_{zb}sin\phi + M_{yb}\ cos\,\phi) \quad (15)$$

Therefore, ψ can be calculated as shown in equation 16. It can be called ψ_{mag_comp} since it is obtained using the data of the magnetometer and complimentary filter (gyroscope and accelerometer).

$$\psi_{mag\_comp} = atan2(sin\psi, cos\psi) \quad (16)$$

From the third line of equation 12, in an ideal situation where ϕ and θ are obtained 100% accurately, M_{zi} should be equal to $M_{zb}\ cos\,\phi\ cos\,\theta\ +\ M_{yb}\ cos\,\theta\ sin\,\phi -\ M_{xb}\ sin\,\theta$. Therefore, the difference between them can be used as an indication of error:

$$E = \left| M_{zi} - (M_{zb} \cos \phi_{Comp} \cos \theta_{Comp} + M_{yb} \cos \theta_{Comp} \sin \phi_{Comp} - M_{xb} \sin \theta_{Comp}) \right| \quad (17)$$

The value of E calculated in equation 17 can be used in real-time to measure the accuracy of the calculations while the system (robot) is operational and in motion. If its value exceeds a certain threshold, it can trigger a calibration or sensor failure alarm. A beneficial feature of this factor is that it is based on the data separately obtained from a magnetometer and a set of accelerometer and gyroscope. Since, in this factor, the terms M_{xb}, M_{yb}, M_{zb} are obtained without using the accelerometer and gyroscope, while ϕ_{Comp} and θ_{Comp} are obtained from the set of accelerometer and gyroscope without using the magnetometer.

In case the value of E exceeds a specific threshold, the value of the coefficient α can be adjusted within it in its normal range of [0.01, 0.07] as an initial solution. If the value of E continues to surpass the threshold, an alarm indicating sensor failure can be triggered.

It is noteworthy that the factor E can be used besides the real-time method mentioned in [2] to detect inaccuracies in the calculated angles and sensor failures.

Note: If the X axis of the inertial frame is towards the South and Z axis is perpendicular to the ground surface, the magnetic field expressed in the inertial frame is $[M_e \cos \alpha \quad 0 \quad M_e \sin \alpha]^T$. Where α is the inclination angle of the Earth's magnetic field and M_e is the magnitude of the Earth's magnetic field at the current point of measurement.

[1] P. Amiri, M. Dayyani, A. Rezaei Lori, A. Boudesh, N. Sina, Fuzzy – Sliding Mode Versus Integral Sliding Mode Controller for a Quadrotor with Mass Uncertainty under Effect of Wind

[2] Diversified Redundancy in the Measurement of Euler Angles Using Accelerometers and Magnetometers

[3] W. T. Higgins. A Comparison of Complementary and Kalman Filtering